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COMMENT

Multiplicity of infinite clusters in percolation above six dimensions

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Abstract. We present Monte Carlo simulations of site percolation near the percolation threshold in five to seven dimensions. In contrast to lower dimensionalities, more than one spanning cluster is found in a system of size L^d for $d > 6$. These results confirm Coniglio's prediction that the breakdown of hyperscaling for $d > 6$ is caused by a multiplicity of infinite clusters at the percolation threshold.

The critical properties of the percolation transition have been widely studied (Essam 1980, Stauffer 1985). It is known that, as the concentration p approaches p_c , the connectedness length diverges and an infinite cluster spanning the whole system appears. It has been generally believed that only one such infinite cluster exists at the percolation threshold. This uniqueness was known to hold in two dimensions (Harris 1960, Fisher 1961). Further, some general heuristic arguments were given (Kikuchi 1970) regarding the existence of a unique spanning cluster of each atomic species in a random alloy of A and B atoms. Only recently Newman and Schulman (1981) rigorously proved that, at the onset of percolation, the only possibilities for the number of infinite clusters were either zero, one or infinity, where, in the latter case, the clusters would have zero density (Aizenman 1985).

Contrary to the numerous computer simulations of percolation in two and three dimensions, there is no numerical evidence for the behaviour of the number of spanning clusters in higher dimensionality. The need for numerical results has become even more striking due to recent discussions and subsequent predictions (Aharony *et al* 1984, Coniglio 1985) on the fractal properties of the infinite cluster above the upper critical dimension d_c . Let us, in fact, consider the mass of this infinite cluster; this is self-similar for length scales $L \ll \xi$, and scales as $M \sim L^D$ with an exponent

$$D = d - \beta/\nu. \quad (1)$$

This fractal dimension D expresses the fact that a density $L^{-\beta/\nu}$ of the total volume L^d of sites belongs to the spanning cluster. By introducing the hyperscaling relation $2 - \alpha = d = 2\beta + \gamma$, it is also possible to rewrite the fractal dimension in (1) as

$$D = (\beta + \gamma)/\nu. \quad (2)$$

From field theory we know that the upper critical dimension for percolation is $d_c = 6$ (Toulouse 1974, Harris *et al* 1975); above this dimension the critical exponents assume their mean field value (i.e. $-\alpha = \beta = \gamma = 1$ and $\nu = \frac{1}{2}$). Therefore for $d > 6$ equations (1) and (2) would lead to two different values for the fractal dimension, $D = d - 2$ and $D = 4$.

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Aharony *et al* (1984) were the first to notice that the breakdown of (1) was caused by the failure of hyperscaling. Subsequently Coniglio (1985) gave a geometrical interpretation to this problem. The hyperscaling relation is in fact equivalent to the assumption that, at the percolation threshold, only a single spanning cluster is present in a volume L^d . Therefore the breakdown of hyperscaling above six dimensions would lead to the conclusion that a critical number of spanning clusters $N_L \sim L^{d-6}$ is present at $p = p_c$, each with a mass that scales as $M \sim L^D$ where $D = 4$. As a consequence the total mass of all the spanning clusters scales as $M_{\text{tot}} \sim N_L L^D \sim L^{d-2}$ (Coniglio 1985) for $d > 6$. Below six dimensions, the usual result $N_L \sim 1$ is recovered and $M = M_{\text{tot}} \sim L^D$.

In this comment we want to test this prediction by Monte Carlo simulation, in order to determine if more than one spanning cluster is present in a system at the percolation threshold in high dimensions. To this end, we consider a system of L^d sites at the nodes of a hypercubic lattice. Each site is present with a probability p and missing with a probability $(1 - p)$. Once each configuration is generated, we proceed to the analysis of the clusters so obtained for a fixed value of p and to the identification of all the possible different clusters spanning the system from the top hyperplane to the bottom one. To do so, we generalise the Hoshen and Kopelman (1976) algorithm to a system in d dimensions with helical boundary conditions. The main advantage of this algorithm, beside being quite efficient ($\sim 3.3 \mu\text{s}/\text{site}$ on the CDC Cyber 76), is that it requires only $L^{d-1} + L^{d-2}$ memory allocations to analyse a system of L^d sites. We then studied systems of different sizes, $L = 3, 4, 5, 6, 20$ ($L = 20$ only in three dimensions), in different dimensions ($d = 3, 5, 6, 7$) and averaged our data over several configurations (respectively 20 000, 3500, 600, 70 configurations for $L = 3, 4, 5, 6$ in $d = 7$). We considered also a range of several p values about the series estimate for the percolation threshold (Gaunt *et al* 1976, Gaunt and Brak 1984). For each system size at a given dimension we monitor at the same time the spanning probability R (the probability that there exists at least one spanning cluster) and the average number of spanning clusters N_L as a function of p . We can then detect a striking difference in the behaviour of N_L below and above $d = 6$.

Whereas N_L is a constant function, approximately equal to one, over the whole range of p and for all system sizes for $d < 6$ (figure 1), N_L exhibits a sharp maximum for $d > 6$ (figure 2). This is located at a value $p^* > p_c^{\text{eff}}(L)$, where $p_c^{\text{eff}}(L)$, the effective percolation threshold at a given L , is identified with the value of p at which $R(p)$ assumes half of its value. As the system size increases, the maximum becomes sharper and moves toward $p_c^{\text{eff}}(L)$, leading to the conclusion that in the limit $L \rightarrow \infty$ the number of infinite clusters will go to infinity at the percolation threshold p_c , and is zero and one for p below and above p_c , respectively, for $d = 7$.

At $d = 6$ the number of spanning clusters N_L has no sharp maximum but shows a weak dependence as a function of p , which could be interpreted as a logarithmic correction at $d = d_c$ (see also Aharony *et al* 1984).

The finite-size scaling analysis for the different system sizes in $d = 7$ is shown in figure 3. A log-log plot of the number of spanning clusters against L gives a value of the critical exponent equal to 1.63 ± 0.30 ; however, by plotting the quantity $\log N_L / \log L$ as a function of $1/\log L$ and extrapolating to the asymptotic value for $L \rightarrow \infty$, we find that the exponent, equal to 1.95 if we consider the data points for $L = 3, 4, 5$, goes down to the given value 1.63 if we also take into account the data for $L = 6$. Therefore, we can conclude that, by analysing larger systems, a lower value of the asymptotic exponent could be obtained, which does not exclude the theoretical prediction $d - 6 = 1$.

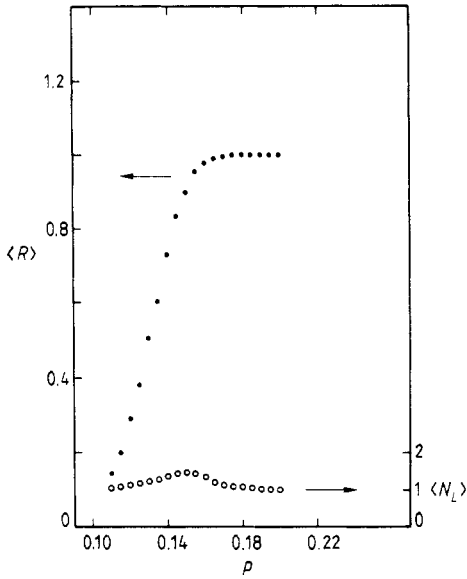


Figure 1. Average spanning probability $\langle R \rangle$ (●, left-hand scale) and average number of spanning clusters $\langle N_L \rangle$ (○, right-hand scale) for 3000 configurations of the system size $L=6$ in $d=5$. The weak dependence of $\langle N_L \rangle$ on p is expected to disappear for larger system sizes.

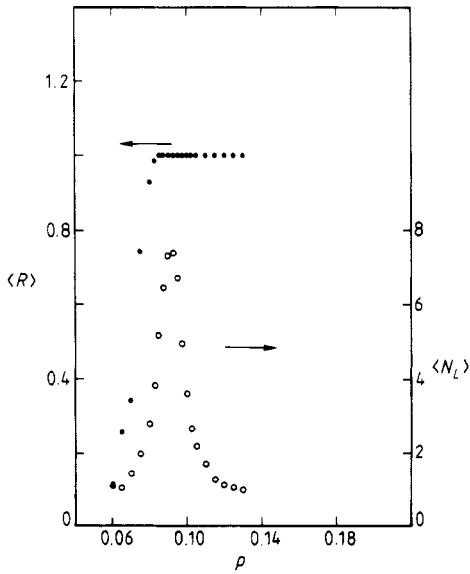


Figure 2. Average spanning probability $\langle R \rangle$ (●, left-hand scale) and average number of spanning clusters $\langle N_L \rangle$ (○, right-hand scale) for 70 configurations of the system size $L=6$ in $d=7$. The three data points about the maximum of the $\langle N_L \rangle$ curve are determined from 270 configurations. The height of the maximum is equal to 7.35 and is located at a value of $p^*=0.0925$.

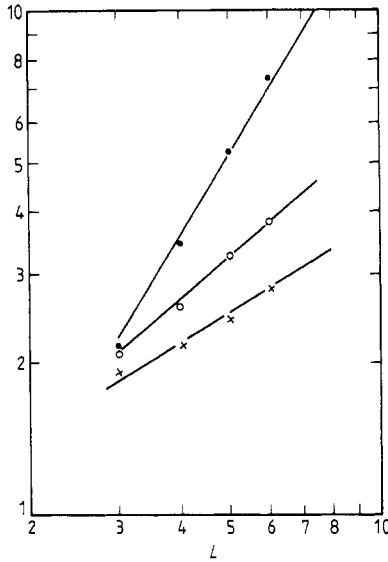


Figure 3. Log-log plot of the average number of clusters $\langle N_L \rangle$ (●), the sixth root of the average mass of a single spanning cluster $\langle M \rangle^{1/6}$ (×) and the sixth root of the average total mass of all spanning clusters $\langle M_{tot} \rangle^{1/6}$ (○) against L . 30 000, 4500, 1000 and 270 configurations are considered for the system sizes $L=3, 4, 5$ and 6 . The values of the slopes are, respectively, equal to 1.63, 0.62 and 0.87.

Furthermore, the mass of the average spanning cluster scales with a fractal dimension $D = 3.74 \pm 0.30$ in seven dimensions, in fairly good agreement with the predicted mean field value $D = 4$. Finally, the total mass of all spanning clusters scales with a fractal dimension $D = 5.19 \pm 0.30$, in good agreement with the theoretical value $D = d - 2 = 5$ (Coniglio 1985).

Therefore, our picture is the following: for $d < d_c$, several large clusters can grow, as long as p is less than p_c , and they can indeed interpenetrate with a relative distance of the order of ξ but, as p reaches the percolation threshold, all these clusters coalesce giving rise to a unique critical cluster spanning the whole system. For $d > d_c$, the N_L spanning clusters are more fragile string-like chains, made of links and dangling ends, and they can interpenetrate each other, without coalescing and keeping a relative distance $\xi_1 \sim \xi^{6/d}$ (Coniglio 1985), even at the percolation threshold.

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